Complete Decentralized Method for On-Line Multi-Robot Trajectory Planning in Well-formed Infrastructures

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Motivation

Industrial intralogistics  Automated transport UAVs

- Individual vehicles must not collide.
Motivation Scenario

Automated warehouse of an e-shop.

A product is ordered through a website:

1. A robot is sent from the depot to the shelf to get the product.

2. The robot carries the product to a human operator, who ships the product.
Problem Definition

- static 2-d environment $\mathcal{W} \subseteq \mathbb{R}^2$
- finite set of endpoints $E \subset \mathcal{W}$
- occupied by $n$ circular mobile robots with radius $r$ and max. speed $v$
- at any time, a robot can be ordered to move from one endpoint to another (assigned a relocation tasks)
- objective: ensure that the destination of each relocation tasks will be reached without collision with other robots
Existing Techniques

New relocation task $s \rightarrow g$ assigned to a robot:

- **Reactive Approach**
  - Follows shortest path from $s$ to $g$ (planned without considering other robots)
  - If potential collision detected – adjust immediate velocity to avoid the collision
  - Collision avoiding velocity computed using ORCA [Van Den Berg et al., 2011].
  - **Not guaranteed** – Collision solved locally, may lead to a deadlock

- **Planning Approach**
  - Interrupt the robots
  - Find coordinated trajectories from current position to destination of each robot
  - All robots start following the found coordinated trajectories
  - Dead-lock free
Circles in a polygonal environment – **NP-hard** [Spirakis and Yap, 1984].

All known complete algorithms are $O(c^n)$ [Ramanathan and Alagar, 1985, Wagner and Choset, 2015, Standley, 2010]

**Figure**: Coordination of Disks [Schwartz and Sharir, 1983]
We characterize a class of environments called *well-formed infrastructures*, where collision-free and dead-lock free trajectory for any relocation task can be computed in polynomial time.
Continuous Best-Response Approach (COBRA)

Decentralized Approach

- Each robot plans its trajectory using its on-board CPU.
- Uses token-based synchronization. The token carries current trajectories of all robots and can be held only by a single robot at a time.

Main idea: When assigned a new relocation task, follow the optimal path to destination that avoids robots that were assigned their relocation tasks earlier.
**COBRA – Algorithm**

When-registered-at \((p)\)

\[
\text{OtherTrajs} \leftarrow \text{request-token}
\]

\[
\text{Traj}_i \leftarrow \text{trajectory that stays at } p \text{ forever}
\]

add \(\text{Traj}_i\) to \(\text{OtherTrajs}\)

release-token \(\text{OtherTrajs}\)

Handle-relocation-task-to \((g)\)

\[
\text{OtherTrajs} \leftarrow \text{request-token}
\]

\[
\text{Traj}_i \leftarrow \text{optimal trajectory to } g \text{ avoiding } \text{OtherTrajs} \tag{1}
\]

update the record of robot \(i\) in \(\text{OtherTrajs}\) with \(\text{Traj}_i\)

release-token \(\text{OtherTrajs}\)

follow \(\text{Traj}_i\)

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\(^1\) Requires a trajectory planner able to find an optimal trajectory for the robot amidst moving obstacles.
COBRA – Completeness

Assumptions:
- static environment
- perfect execution of trajectories
- non-interruptible relocation tasks

Theorem (completeness in well-formed infrastructures)

If COBRA is used to coordinate relocation tasks between endpoints of a well-formed infrastructure, then all relocation tasks will be carried out without collision.
Well-formed Infrastructure

**Definition (infrastructure)**

An infrastructure is a tuple $\langle \mathcal{W}, P \rangle$, where

- $\mathcal{W} \subset \mathbb{R}^d$ is a set of obstacle-free positions (free space)
- $P \subset \mathcal{W}$ is a set of distinguished locations called endpoints

- robots move only between endpoints
- endpoints represent workplaces, parking places, etc.

**Definition (well-formed infrastructure)**

An infrastructure $\langle \mathcal{W}, P \rangle$ is called **well-formed** for robots with max. radius $r$ if there exists a path between any two endpoints $p_1$ and $p_2$ that avoids obstacles with $r$-clearance and all other endpoints with $2r$-clearance.
Well-formed Infrastructure – Example

Well-formed infrastructure

Ill-formed infrastructure
(There is no path between \( e_1 \) and \( e_4 \) that avoids \( e_3 \) with \( 2r \)-clearance)
Well-formed Infrastructures in Real-world

Human made environments are usually structured as well-formed infrastructures:
Theorem (polynomial complexity)

The worst-case asymptotic complexity of a single relocation task handling using COBRA with time-extended roadmap planning is $O(n^2)$, where $n$ is the number of robots in the system.
Results (Office Corridor)

- Avg prolongation of relocation task (avg. 24s long) due to collision avoidance
- Success rate

Method: COBRA ORCA
Illustration: COBRA in Office Corridor

(click to play)
Results (Warehouse)

Avg prolongation of relocation task (avg. 32s long) due to collision avoidance

Method: COBRA ORCA

Success rate

Instances solved [%]
Existing methods for collision avoidance in multi-robot systems are either
   a) prone to deadlocks or
   b) intractable.

We characterized class of environments called **well-formed infrastructures** and designed and
**polynomial guaranteed method COBRA** that can be used for trajectory coordination in such environments.

Benchmark instances and Java implementation available at:

http://agents.cz/~cap/
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Questions?


